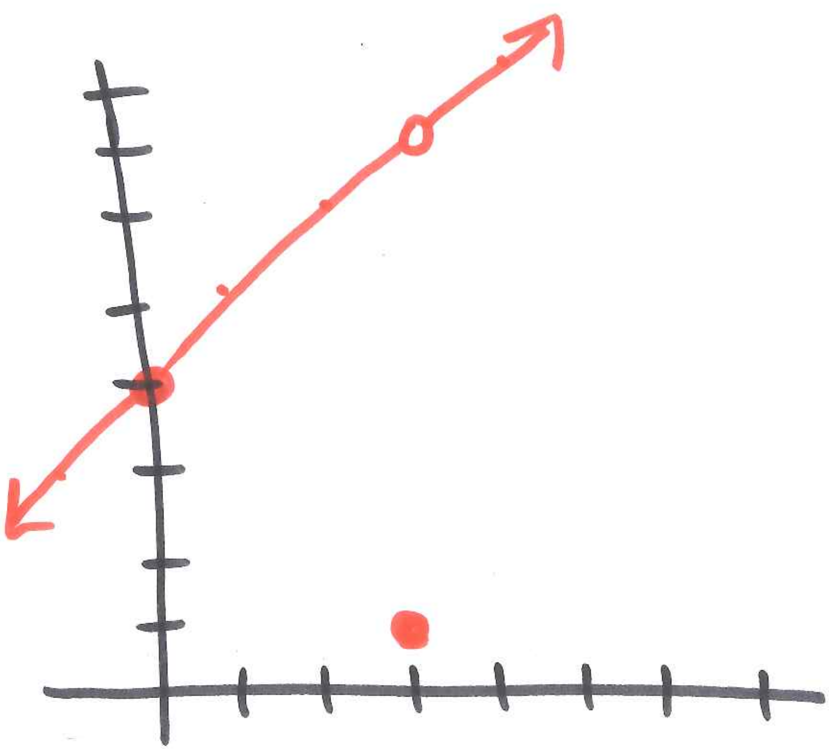
Chapter 3 - Day 2

Review:

<u>Ex</u>: let $g(x) = \begin{cases} x+4 & \text{for } x \neq 3 \\ 1 & \text{for } x = 3 \end{cases}$

find $\lim_{x \to 3} g(x)$



$\lim_{x \to 3} g(x) = 7$

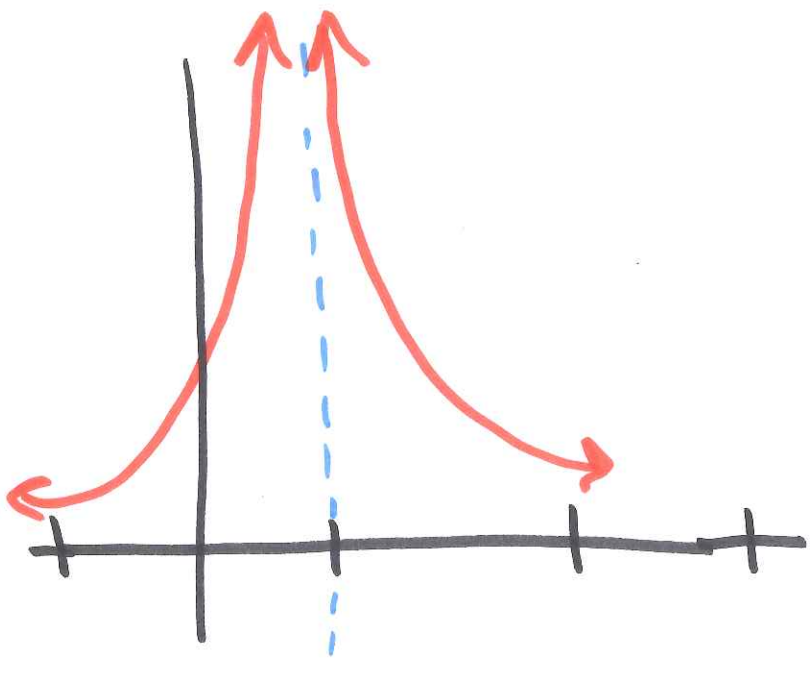
This class we'll focus on "What do we do when we have division by zero in a limit?"

<u>Case 1</u>: Use substitution and the result is $\frac{a}{0}$ where $a \neq 0$.

This expression goes on without bound. We say this limit does not exist and write DNE

Ex: find $\lim_{x \to 1} \frac{5}{(x-1)^2}$

let $f(x) = \frac{5}{(x-1)^2}$



x	$f(x)$
.9	500
.99	50,000
.999	5,000,000
1.001	5,000,000
1.01	50,000
1.1	500

increasing without bound

thus $\lim_{x \to 1} \frac{5}{(x-1)^2} = \text{"}\infty\text{"} = \text{DNE}$

*Note: when you plug in 1,

$\frac{5}{(1-1)^2} = \text{"}\frac{5}{0}\text{"}$ thus $\lim_{x \to 1} \frac{5}{(x-1)^2} = \text{DNE}$

Ex: Analyze the limit

$$\lim_{x \to 0} \frac{4}{\sqrt{x}}$$

*Note: $f(x) = \frac{4}{\sqrt{x}}$ has domain $(0,\infty)$ so this is really asking

$$\lim_{x \to 0^+} \frac{4}{\sqrt{x}}$$

Plug in 0 → $\frac{4}{\sqrt{0}} = \frac{4}{0}$

thus $\lim_{x \to 0} \frac{4}{\sqrt{x}} = \text{DNE}$

__Case 2__: Use substitution and the result is $\frac{0}{0}$.

$\frac{0}{0}$ gives us no information about the limit. It only tells us we need to do more work.

__Ex__: Analyze $\lim\limits_{x \to 0} \frac{3x}{x}$

*Note: $\frac{3\cancel{x}}{\cancel{x}} = 3$ for $x \neq 0$
(and we aren't at 0, merely near it)

thus

$$\lim_{x \to 0} \frac{3x}{x} = \lim_{x \to 0} 3 = \boxed{3}$$

Ex: Analyze $\lim_{x \to 5} \frac{x^2 - x - 20}{x - 5}$

"$\frac{0}{0}$" $\frac{x^2 - x - 20}{x - 5} = \frac{\cancel{(x-5)}(x+4)}{\cancel{(x-5)}} = x+4$

thus

$\lim_{x \to 5} \frac{x^2 - x - 20}{x - 5} = \lim_{x \to 5} x+4 = 5+4 = \boxed{9}$

Ex: find the limit $\lim_{h \to 0} \frac{(h-2)^2 - 4}{h}$

"$\frac{0}{0}$" $\frac{(h-2)^2 - 4}{h} = \frac{h^2 - 4h + \cancel{4} - \cancel{4}}{h} = h - 4$

$\lim_{h \to 0} \frac{(h-2)^2 - 4}{h} = \lim_{h \to 0} h-4 = 0-4 = \boxed{-4}$

Ex: Consider the limits

- $\lim\limits_{x \to 0^+} \frac{|x|}{x}$

"$\frac{0}{0}$" x positive so $\frac{|x|}{x} = \frac{x}{x} = 1$

so $\lim\limits_{x \to 0^+} \frac{|x|}{x} = \lim\limits_{x \to 0^+} 1 = \boxed{1}$

- $\lim\limits_{x \to 0^-} \frac{|x|}{x}$

"$\frac{0}{0}$" x negative so $\frac{|x|}{x} = \frac{-x}{x} = -1$

so $\lim\limits_{x \to 0^-} \frac{|x|}{x} = \lim\limits_{x \to 0^-} -1 = \boxed{-1}$

- $\lim\limits_{x \to 0} \frac{|x|}{x}$

because $\lim\limits_{x \to 0^+} \frac{|x|}{x} \neq \lim\limits_{x \to 0^-} \frac{|x|}{x}$

then $\lim\limits_{x \to 0} \frac{|x|}{x} = \text{DNE}$

limits at infinity tell us about the end behavior of a rational function.

<u>Theorem</u>: for $p(x)$ and $q(x)$ polynomials

$$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} =$$

$$\lim_{x \to \pm\infty} \frac{\text{highest order term of } p(x)}{\text{highest order term of } q(x)}$$

<u>Ex</u>: Calculate $\displaystyle\lim_{x \to -\infty} \frac{6x^2}{x^3+1}$

$$\lim_{x \to -\infty} \frac{6x^2}{x^3+1} = \lim_{x \to -\infty} \frac{6x^2}{x^3}$$

cancel x^2

$$= \lim_{x \to -\infty} \frac{6}{x} = \text{"}\frac{6}{-\infty}\text{"} = \boxed{0}$$

Ex: Calculate $\lim\limits_{x\to\infty} \dfrac{(3x-1)^2}{5x^2+8}$

$$\lim_{x\to\infty} \frac{(3x-1)^2}{5x^2+8} = \lim_{x\to\infty} \frac{9x^2}{5x^2}$$ cancel x^2

$$= \lim_{x\to\infty} \frac{9}{5} = \boxed{\frac{9}{5}}$$

Ex: Calculate $\lim\limits_{x\to\infty} \dfrac{(2x)^2(5x+1)}{(2x+1)(3x)(x-4)}$

$$\lim_{x\to\infty} \frac{(2x)^2(5x+1)}{(2x+1)(3x)(x-4)} = \lim_{x\to\infty} \frac{20x^3}{6x^3}$$

cancel x^3 $$= \lim_{x\to\infty} \frac{20}{6} = \frac{20}{6} = \boxed{\frac{10}{3}}$$